

Bayesian Data Analysis (PHY/CSI/INF 451/551)

HW#2w

1. Independent Pair of Fair 6-Sided Dice.

Consider an independent pair of fair 6-sided dice with sides indexed by i and j .

a. Since they are independent, how does $p(i | j, I)$ relate to $p(i | I)$?

b. What is the probability of rolling $i=2$ on the first 6-sided die? That is, what is $p(i = 2 | I)$?

c. What quantity does $p(i = 2, j = 4 | I)$ represent? And what is its value?

d. What is the average value (also called the expected value) of i ?

e. Is it possible to ever observe this expected value? Why or why not?

f. What is the expected value of $i+j$?

g. What is the most probable value of $i+j$?

2. Imagine that we have a pair of six-sided dice that are attached to one another with a string. Below is a partially-filled table of the probabilities for rolling different values indexed by i and j .

$P(i,j I)$	$j = 1$	$j = 2$	$j = 3$	$j = 4$	$j = 5$	$j = 6$
$i = 1$	0	0	0	0	0	0
$i = 2$	0		$\frac{2}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	0
$i = 3$	0	$\frac{2}{36}$	$\frac{4}{36}$	$\frac{1}{36}$	$\frac{2}{36}$	0
$i = 4$	0	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{2}{36}$	0
$i = 5$	0	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{4}{36}$	0
$i = 6$	0	0	0	0	0	0

a. Fill in the missing entry in the table above.

b. What is the expected value of $i+j$?

c. What is the probability $p(i | I)$ for all values of i ?

$$p(i = 1 | I) =$$

$$p(i = 2 | I) =$$

$$p(i = 3 | I) =$$

$$p(i = 4 | I) =$$

$$p(i = 5 | I) =$$

$$p(i = 6 | I) =$$

d. What is the expected value of i ?

3. In the problem #1, we found that $p(i | j, I) = p(i | I)$ when i and j are independent.

Prove (algebraically using the sum and/or the product rules) that if

$p(x, y | I) = p(x | I) p(y | I)$ then $p(x | y, I) = p(x | I)$ meaning that if x and y are independent, then you can just drop any conditioning of x on y .

4. Rolling 6's.

a. Imagine that you have **one set of 6** six-sided dice. Calculate the probability that you will **roll at least one 6** if you roll all six dice.

b. Consider that you have **two sets of 6** six-sided dice (12 dice total). Calculate the probability that you will **roll at least two 6's** if you roll all 12 dice.

c. Consider that you have **three sets of 6** six-sided dice (18 dice total). Calculate the probability that you will **roll at least three 6's** if you roll all 18 dice.

d. Are the probabilities in parts a-c equal? Why or why not? And how do these results compare to the simulations you performed in homework set HW1p?