Bayesian Data Analysis
PHY/CSI/INF 451/551
HW 4w
Show all work

The following problems rely on Bayes Theorem
Explicitly list the hypotheses and data involved as well as the prior probabilities and likelihoods.

1. Crime Scene
A crime has been committed. It is known that the crime was committed by exactly one person and that there are 1000 people who could have committed the crime. In the absence of evidence each of these 1000 people are equally likely to have committed the crime.

Blood was found at the scene of the crime and it is apparent that the criminal cut themself while committing the crime. The blood is analyzed and is found to be Blood Type O-positive. It is known that 38% of the population has O-positive type blood.

A suspect S is apprehended and it is found that this person has Blood Type O-positive.

Given the evidence, what is the probability that person S committed the crime?
Will more evidence be needed to ensure that they have arrested the right person?

The following problems rely on Bayes Theorem
Explicitly list the hypotheses and data involved as well as the prior probabilities and likelihoods.

2. Crime Scene
Consider the crime in Problem #1, above.
However, this time blood was found at the scene of the crime and it is apparent that the criminal cut themself while committing the crime. The blood is analyzed and is found to be Blood Type B-negative. It is known that only 2% of the population has B-negative type blood.

A suspect S is apprehended and it is found that this person has Blood Type B-negative.

Given the evidence, what is the probability that person S committed the crime?
Will more evidence be needed to ensure that they have arrested the right person?

3. Parts Suppliers
Two different suppliers, A and B, provide a manufacturer with the same part. All supplies of this part are kept in a large bin. In the past, 3% of the parts supplied by A and 7% of the parts supplied by B have been defective. A supplies four times as many parts as B.

Suppose you reach into the bin and select a part, and find it is non-defective.
What is the probability that it was supplied by A?

4. You are a laptop repair person. When a laptop stops working, it is due to a failed power supply 30% of the time. If a laptop's power supply has failed, there is a 45% probability that plugging it in will produce smoke. If a laptop's power supply is OK, but something else is wrong, there is only a 5% chance that plugging it in will produce smoke. A customer brings you a malfunctioning laptop. You plug in the laptop and find that it produces smoke. What is the probability that a smoke-producing laptop has a failed power supply?

5. The blue M&M candies were introduced in 1995. Before then, the color mix in a bag of plain M&Ms was (30% Brown, 20% Yellow, 20% Red, 10% Green, 10% Orange, 10% Tan). Afterward it was (24% Blue, 20% Green, 16% Orange, 14% Yellow, 13% Red, 13% Brown).

A friend of mine has two bags of M&Ms, and she tells me that one is from 1994 and one from 1996. She won't tell me which is which, but she gives me one M&M from each bag. One is yellow and one is green. What is the probability that the yellow M&M came from the 1994 bag?

This problem is more challenging because one has two pieces of data (a yellow M&M and a green M&M).
We have a joint likelihood, which must be factored using the product rule.