Bayesian Data Analysis (PHY/CSI/INF 451/551)
HW#2w

1. Independent Pair of Fair 6-Sided Dice.
Consider an independent pair of fair 6-sided dice with sides indexed by i and j.

a. Since they are independent, how does $p(i \mid j, I)$ relate to $p(i \mid I)$?

b. What is the probability of rolling $i=2$ on the first 6-sided die? That is, what is $p(i = 2 \mid I)$?

c. What quantity does $p(i = 2, j = 4 \mid I)$ represent? And what is its value?

d. What is the average value (also called the expected value) of i ?

e. Is it possible to ever observe this expected value? Why or why not?

f. What is the expected value of $i+j$?

g. What is the most probable value of $i+j$?

2. Imagine that we have a pair of six-sided dice that are attached to one another with a string. Below is a partially-filled in table of the probabilities for rolling different values indexed by i and j.

$P(i,j\|I)$	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$
$i=1$	0	0	0	0	0	0
$i=2$	0		$\frac{2}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	0
$i=3$	0	$\frac{2}{36}$	$\frac{4}{36}$	$\frac{1}{36}$	$\frac{2}{36}$	0
$i=4$	0	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{2}{36}$	0
$i=5$	0	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{4}{36}$	0
$i=6$	0	0	0	0	0	0

a. Fill in the missing entry in the table above.

b. What is the expected value of i+j ?

c. What is the probability p(i | I) for all values of i ?

$p(i=1\,|\,I)=$

$p(i=2\,|\,I)=$

$p(i=3\,|\,I)=$

$p(i=4\,|\,I)=$

$p(i=5\,|\,I)=$

$p(i=6\,|\,I)=$

d. What is the expected value of i ?

3. In the problem #1, we found that $p(i \mid j, I) = p(i \mid I)$ when i and j are independent.

Prove (algebraically using the sum and/or the product rules) that if $p(x, y \mid I) = p(x \mid I)\, p(y \mid I)$ then $p(x \mid y, I) = p(x \mid I)$ meaning that if x and y are independent, then you can just drop any conditioning of x on y.

4. Rolling 6's.

a. Imagine that you have **one set of 6** six-sided dice. Calculate the probability that you will **roll at least one 6** if you roll all six dice.

b. Consider that you have **two sets of 6** six-sided dice (12 dice total). Calculate the probability that you will **roll at least two 6's** if you roll all 12 dice.

c. Consider that you have **three sets of 6** six-sided dice (18 dice total). Calculate the probability that you will **roll at least three 6's** if you roll all 18 dice.

d. Are the probabilities in parts a-c equal? Why or why not?