Bayesian Data Analysis (PHY/CSI/INF 451/551)
HW#3p

1. Generate a vector of 1000 uniformly-distributed samples (numbers between zero and one), and plot them in a histogram to verify that they are uniformly distributed.

2. Obtain 100 uniformly-distributed samples between 0.3 and 0.5 and plot their distribution in a histogram.

3. Use the MATLAB function randn to generate 1000 normally distributed samples with zero mean and a standard deviation of one. Plot their distribution in a histogram.

4. The normal distribution, also known as a Gaussian distribution, is a bell-shaped curve. Mathematically, the distribution is

   $$P(x \mid \mu, \sigma, I) = \frac{1}{\sqrt{2\,\pi\,\sigma^2}}\, exp\left(-\frac{(x-\,\mu)^2}{2\,\sigma^2}\right).$$

   Plot the Gaussian curve (for $\mu = 0$ and $\sigma = 1$) over the histogram above (#3), showing both results in the same figure, and show that the histogram indeed follows the Gaussian distribution.

5. Write code to run the following simulation. Define, and plot, a rectangle going from $x = 0$ to $x = 4$ on the x-axis and from $y = 0$ to $y = 4$ on the y-axis. Define, and plot, a function that has the following behavior

   $$p(x) = \begin{cases} 1 & for \quad 0 \le x < 1 \\ 2 & for \quad 1 \le x < 2 \\ 3 & for \quad 2 \le x < 3 \\ 4 & for \quad 3 \le x < 4 \end{cases}$$

   Next sample 1000 points uniformly distributed within this rectangle. You can do this by uniformly sampling the x coordinates of the points in the range $0 \le x < 4$, and the same for the y-coordinates.

   Plot these 1000 points on the same figure, coloring the points above the staircase function $p(x)$ red, and the ones below $p(x)$ blue.

   Last, take the blue samples, the ones below the function $p(x)$, and on a separate figure make a histogram of their x values. Show that this histogram is the staircase function $p(x)$.
   Note that you have ignored (rejected) the red samples that lie above the function $p(x)$.

   This technique allows you to draw samples with any distribution $p(x)$!

EXTRA CREDIT

Go through #5 again and try another (your choice) function for $p(x)$, and include the two figures (rectangle with $p(x)$ and samples AND the histogram of those samples) in your homework for extra credit.