Bayesian Data Analysis (PHY/CSI/INF 451/551)
HW#4w

1. The Art Show

Three promising artists have entered an art show from which one piece will be awarded First Place.

Pamela submitted 15 paintings to the show. In the past, 4% of the time she has been awarded First Place. Patricia submitted 5 paintings to the show. In the past, 6% of the time she has been awarded First Place. Pablo submitted 10 paintings to the show. In the past, 3% of the time he has been awarded First Place.

Find the probability that Pamela will be awarded First Place.

2. Cheating in a Roll-Playing Game?!?

You are playing a roll-playing game with friends, and the Game Master preceding over the game has to roll two TWENTY (20) – sided dice to determine the fate of you and your friend. The Game Master rolls the dice behind a screen and claims that he rolled a 2 and a 9. Seeing that you were hoping for rolls above 16, you are suspicious that he rolled two TWELVE (12) – sided dice by mistake (or on purpose?).

Given that two dice were rolled and resulted in a 2 and a 9, what is the probability that the Game Master rolled two TWELVE-sided dice, as opposed to two TWENTY-sided dice?

3. Ashley has asked a professor for a recommendation for graduate school.

She estimates that the probability that the letter will be strong is 50%, the probability that the letter will be weak is 20%, and the probability that the letter will be mediocre is 30%.
She also estimates that if the letter is strong, the probability that she will be accepted into the graduate program is 80%; if it is mediocre there is only a 20% chance that she will be accepted, and if it is weak there is a 5% chance of being accepted.

Given that Ashley was accepted into the graduate program, what is the probability that the letter was strong?

4. **An archeologist is studying drug abuse** in the times of the late Roman Empire (circa 250–450 A.D.). This is accomplished by testing pottery for traces of opium. Samples from one such vessel were sent to five different labs to determine the age of the artifact.

Laboratory A returned a report stating that they dated the artifact at 327.1 ± 1.2 A.D.
Laboratory B returned a report stating that they dated the artifact at 332.5 ± 3.2 A.D.
Laboratory C returned a report stating that they dated the artifact at 321.2 ± 7.5 A.D.

Laboratory D returned a report stating that they dated the artifact at 318.0 ± 2.2 A.D.
Laboratory E returned a report stating that they dated the artifact at $325.3 \ \pm 3.5$ A.D.

Assuming that the listed uncertainties represent a 1-σ estimate from a Gaussian distribution:

A. Compute the most probable estimate for the age of the pottery and its uncertainty.
You might consult with the Measuring Lengths lecture notes, but you will have to complete the notes by deriving the formula for estimating the uncertainty of the most probable estimate.

B. Plot the data with error bars along with lines illustrating the most probable age along with the uncertainties.

5. **Google has a team of computer scientists studying client mouse-clicking behavior** with the aim of modeling where on a web page people are more likely to click. (While Google actually studies this, the remainder of this problem is fictional.) Consider a web page that has a visual focal point somewhere near the center of the page. Say that there is a uniform probability that someone will click within a circle of radius R from the focus. That is,

$$p(r, \theta \mid I) = C \qquad \text{for} \qquad r \leq R \ \text{(0 otherwise)}$$

Keep in mind that the differential area is $r\, dr\, d\theta$. So that the total probability is found by integrating $\int_0^R \int_0^{2\pi} p(r, \theta \mid I)\, r\, dr\, d\theta$.

A. Find an expression for the constant C in terms of the radius R. That is, find C so that the probability, $p(r, \theta \mid I)$ above, is properly normalized.

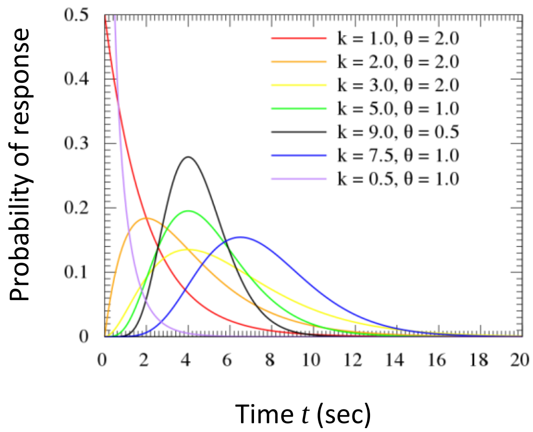
B. Find an expression describing the probability that a client will click on something located a distance r from the focus. That is, find an expression for $p(r \mid I)$.

C. If $R = 6$cm, find the probability that a client will click on something located between 2cm and 3cm away from the focus. That is, $2cm \ \leq \ r \ \leq 3cm$.

6. In class we have worked mainly with Gaussian distributions, which are symmetric. This is not always representative of reality. **Modern functional Magnetic Resonance Imaging (MRI) relies on the Blood Oxygenation Level Dependent (BOLD) response**, in which one can track the rush of oxygenated blood to brain regions that are active during an experiment. The timing of the BOLD response has been found to be well-modeled by a Gamma distribution (Knuth, Ardekani, Helpern; 2001):

$$p(t|I) = \frac{t^{k-1}}{\Gamma(k)}\, \theta^{-k}\, e^{-\frac{t}{\theta}}$$

in which t is the elapsed time after physical stimulation, k and θ are parameters, and $\Gamma(.)$ is the Gamma function (which you will not need to know).



A. Find and expression for the most probable time, $\hat{t}$, of the BOLD response in terms of the parameters k and θ. (The graph above may be useful in checking your results.)

B. Find an expression for the uncertainty, σ_t, associated with the time $\hat{t}$ in terms of the parameters k and θ.