Measuring the Verdet constant of flint glass

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Abstract

The Magneto-Optical Faraday Effect is the change in the angle of polarization of light traveling through a medium where a magnetic field is present. The property of the medium that determines the angle change is the Verdet constant, and it was measured for a sample of flint glass in the presented lab. The glass was placed between two solenoids, which had been previously calibrated, and a polarized red laser beam was passed through it. The change in the beam’s polarization angle as the magnetic field varied was determined by measuring the beam’s intensity after it passed through a polarization filter, and measuring how much rotation the laser needed to restore the measured intensity to its previous value. This was done for ten different field strengths and the corresponding Verdet constants for each were averaged to give the final result of $2.257 \pm 0.046 \text{ rad/kilogauss\cdot meter}$ for the Verdet constant of the flint glass.

In the units above, the units should use abbreviations, and do not use an asterisk for multiplication. Instead write this: $\text{rad/kG\cdot m}$ or $\text{rad/kG\cdot m}$.

Introduction

The Magneto-Optical Faraday Effect occurs when polarized light passes through a medium and a magnetic field is parallel to the direction of light propagation. The light’s angle of polarization will then gradually rotate as it passes through the material at a rate that is proportional to both the magnetic field strength and the Verdet constant, which is specific to the material and frequency. Specifically, the change in the angle of polarization $\Delta \theta$ as a result of this effect is given by
\[ \Delta \theta = B L V. \]  \hspace{1cm} (1)

Where \( B \) is the strength of the magnetic field component parallel to propagation, \( L \) is the length of the material, and \( V \) is the Verdet constant. The Verdet constant of a material can then be determined from

\[ V = \frac{\Delta \theta}{\Delta B L} \] \hspace{1cm} (2)

where \( \Delta B \) represents the change in the magnetic field strength.

The Magneto-Optical Faraday Effect results when the external magnetic field causes atoms in the medium to undergo Larmor precession. This results in a different index of refraction for left and right circular polarized light. Plane polarized light is composed evenly of both right and left circular polarized light, and the relative phase of both components determines the angle of polarization. If the two components have different indexes of refraction, they will have different wavelengths, and thus their relative phase will gradually change, changing the angle of polarization.

Uncertainty estimates in this report were calculated in the following way: if \( f \) is a function of an unspecified number of arguments \( a, b, ... c \), then the uncertainty \( \sigma_f \) of \( f \) resulting from the uncertainties of \( a, b, ... c \) is given by [2]

\[ \sigma_f = \sqrt{\sigma_a^2 \left( \frac{\partial f}{\partial a} \right)^2 + \sigma_b^2 \left( \frac{\partial f}{\partial b} \right)^2 + \cdots + \sigma_c^2 \left( \frac{\partial f}{\partial c} \right)^2}. \] \hspace{1cm} (3)

One consequence of (3) is that uncertainties are added in quadrature.

**Experimental design**

The experimental setup required a transparent medium placed in a magnetic field. A block of flint glass was chosen as the medium and it was held in place with a clamp between two hollow solenoids which allowed the laser beam to pass through them. The solenoids were connected in series to an adjustable power supply and amp meter which measured their current. The
solenoids produced a magnetic field in the glass with a strength which could be determined from the amp meter once it was calibrated to the magnetic field in the gap.

Polarized light also needed to pass through the glass, and the change in its polarization angle needed to be measured. A rotatable red laser was attached to the end of one solenoid and it produced a polarized red beam that passed through both solenoids as well as the glass between them. It then ended at a rotatable device attached to the end of the other solenoid. This device consisted of a polarized filter, a diffuser, and then a photo-resistor. The photo-resistor was connected to a microcontroller which measured and displayed the change in the photo-resistor current, and thus indicated the beam intensity. When the beam’s angle of polarization changed, a different percentage passed through the polarized filter and produced a different reading from the microcontroller.

The laser was rotated with a lever which had a second laser attached its end. The second laser produced a beam which was perpendicular to the axis of rotation and made a spot on a piece of paper which was attached to a vertical board. The board was parallel to that axis of rotation. This allowed the rotation of the first mentioned laser to be precisely measured from the change in the laser spot.

In order to calibrate the amp meter to the strength of the magnetic field, a gauss-meter probe was placed between the solenoids and held in place with a clamp. The current in the solenoids was increased ten increments, with each increase roughly corresponding to a ten percent increase in magnetic field strength. For each increment the amperage and magnetic field were both measured. The magnetic field for each increment was measured at three points: one in the center, and two at both edges of the gap between the solenoids. The readings were averaged, and this value was assigned to the amp meter reading. The uncertainty assigned to the magnetic field was calculated by dividing the standard deviation of the three measurements by $\sqrt{3}$. The calibration measurements are displayed in Figure 3.

After calibration, the flint glass was placed between the solenoids. The laser angle was set so that the beam from the second laser was horizontal, and orthogonal to the board that the paper was attached to. The laser spot on the paper was marked and the current reading from
the photo resistor was recorded. The solenoid current was then increased to the maximum current the amp meter was calibrated to. This resulted in a change in the beam intensity at the photo resistor. The first laser was then rotated until the photo resistor current was restored to its original value, making the laser rotation angle equal and opposite the angle change caused from the Faraday effect. The new laser spot was marked and the solenoid current was decreased to the second highest calibrated current. The lever was again moved until the photo resistor value was restored, and the new laser spot was marked. This process was repeated until measurements were taken at all calibration points. The experimental setup is displayed in Figure 1.

![Experimental setup](image)

**Figure 1.** The experimental setup containing solenoids and positioned glass, along with amp meter, gauss meter, and microprocessor.

The distance between two laser spots $dx$ was used to measure the corresponding angle change $d\theta$ with equation

$$d\theta = \arctan \left( \frac{dx}{D} \right) \quad (4)$$

where $D$ was the distance between the axis of rotation and the board. $D$ was measured by measuring the distance from the board to the edge of a cylinder centered on the axis of rotation with a tape measure, and then the diameter of a cylinder from another identical apparatus was measured separately to obtain the cylinder radius. Both of the uncertainties
were added in quadrature to get the uncertainty of \( D \). Using equation (3), the uncertainty of \( \theta \) was calculated with equation

\[
\sigma_\theta = \sqrt{\frac{\sigma_{d\chi}^2 + \sigma_D^2}{1 + \left( \frac{d\chi}{D} \right)^2}}.
\]  

\[ (5) \]

**Experimental results**

The change in polarization angle was measured for ten incremental magnetic field strengths. The Verdet constant for each angle measurement was calculated from equation (2). Using equation (3), the uncertainties of \( \theta, L, \) and \( B \), produced the uncertainty of the Verdet constant \( \sigma_V \) with equation

\[
\sigma_V = \sqrt{\sigma^2_\theta \left( \frac{1}{\Delta BL} \right)^2 + \sigma^2_B \left( \frac{L \Delta \theta}{(\Delta BL)^2} \right)^2 + \sigma^2_L \left( \frac{\Delta B \Delta \theta}{(\Delta BL)^2} \right)^2}.
\]

\[ (6) \]

A weighted average of all Verdet constants \( \langle V \rangle \) was calculated with equation²

\[
\langle V \rangle = \sum \frac{V_i}{\sigma^2_V} \div \sum \frac{1}{\sigma^2_V}.
\]

\[ (7) \]

and the uncertainty \( \sigma_{\langle V \rangle} \) was calculated with equation²

\[
\frac{1}{\sigma_{\langle V \rangle}} = \sqrt{\sum \frac{1}{\sigma^2_V}}.
\]

\[ (8) \]

The width of the flint glass \( L \) was obtained from the manufacturer, and was 2 \( \pm \) 0.05cm. The measured values for current, magnetic field, angle displacement, and individual and averaged Verdet constants are in Table 1. A graph of angle displacements vs magnetic field strengths is displayed in Figure 2, and shows a linear relationship.
Table 1. Measured angle displacement for ten magnetic field strengths, their calculated Verdet constants, and average Verdet constant. Along with corresponding uncertainties and solenoid current.

<table>
<thead>
<tr>
<th>Current (amps)</th>
<th>Magnetic field B (kilo-gauss)</th>
<th>σ_B</th>
<th>Δθ (rad)</th>
<th>σ_θ</th>
<th>Verdet constant V (rad/kilo-gauss*meter)</th>
<th>σ_V</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.58</td>
<td>0.948</td>
<td>0.0194</td>
<td>0.03927</td>
<td>0.00067</td>
<td>2.12</td>
<td>0.078</td>
</tr>
<tr>
<td>1.42</td>
<td>0.858</td>
<td>0.0309</td>
<td>0.03726</td>
<td>0.00067</td>
<td>2.23</td>
<td>0.126</td>
</tr>
<tr>
<td>1.24</td>
<td>0.751</td>
<td>0.0199</td>
<td>0.03256</td>
<td>0.00067</td>
<td>2.23</td>
<td>0.124</td>
</tr>
<tr>
<td>1.14</td>
<td>0.680</td>
<td>0.0079</td>
<td>0.03088</td>
<td>0.00067</td>
<td>2.35</td>
<td>0.094</td>
</tr>
<tr>
<td>0.97</td>
<td>0.578</td>
<td>0.0097</td>
<td>0.02753</td>
<td>0.00067</td>
<td>2.48</td>
<td>0.146</td>
</tr>
<tr>
<td>0.82</td>
<td>0.506</td>
<td>0.0138</td>
<td>0.02551</td>
<td>0.00067</td>
<td>2.64</td>
<td>0.284</td>
</tr>
<tr>
<td>0.60</td>
<td>0.396</td>
<td>0.0132</td>
<td>0.01880</td>
<td>0.00067</td>
<td>2.51</td>
<td>0.516</td>
</tr>
<tr>
<td>0.41</td>
<td>0.281</td>
<td>0.0105</td>
<td>0.01209</td>
<td>0.00067</td>
<td>2.33</td>
<td>1.023</td>
</tr>
<tr>
<td>0.32</td>
<td>0.225</td>
<td>0.0078</td>
<td>0.00873</td>
<td>0.00067</td>
<td>2.15</td>
<td>1.331</td>
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<tr>
<td>0.12</td>
<td>0.096</td>
<td>0.0037</td>
<td>0.00201</td>
<td>0.00067</td>
<td>1.37</td>
<td>4.474</td>
</tr>
<tr>
<td>0.00</td>
<td>0.022</td>
<td>0.0019</td>
<td>0.00000</td>
<td>0.00067</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Weighted average of V: 2.257
Uncertainty of V: 0.046
The final measurement for the flint glass Verdet constant was $2.257 \pm 0.046 \text{ rad kilogauss}^{-1} \text{ meter}$. This was 26% greater than the value obtained by Sayan [1]. Possible sources of error not accounted for include possible changes in intensity at the photo-resistor not caused by the polarization filter. This could have possibly resulted from changes in the beam propagation...
angle when it rotated, or from unevenness on the flint glass surface that could diffuse or bend the beam if its position or polarization angle changed slightly. The magnetic field in the glass also varied greatly over space, and neither uncertainties in the three individual field measurements nor variations in the probe positions between different calibration points were accounted for.

Conclusion

The Verdet constant for a sample of flint glass was measured by placing the glass between two previously calibrated solenoids, passing a red polarized laser beam through the glass, measuring the beam intensity after it passed through a polarization filter, rotating the laser so that the intensity reading remained unchanged by the change in the magnetic field, and measuring the rotation angle of the laser. Angle measurements were taken for ten different magnetic field strengths, and a weighted average of those measurements’ Verdet constants gave the final Verdet constant as $2.257 \pm 0.046 \frac{\text{rad}}{\text{kgauss} \cdot \text{m}}$.

Acknowledgments

Thank you to XXXX and Kevin Wynne for their help in performing the experiment as well as the information they gave.

References

2. *If You Are Uncertain About Uncertainty*, Class handout, State University of New York (February 2017)

*Note that Reference 3 above is not mentioned in the text (all references should be mentioned in the text, just like all the figures and tables).*

*In addition to citing the first source as [1], you could write (Sayan 1997). The latter method is more difficult for the other two sources, which don’t have the author names. You could write*
(SUNY 2017a) and (SUNY 2017b), but that is a little messy. It might be best to stick to [2] and [3].